Preface to the digitalized version (2018) of
Differential Geometry Without Real Numbers".

This is just a scanned and digitalized version of the "Second Edition", which was printed and circulated in 1980, shortly after the first edition 1979, almost 40 years ago. This was at a time when the term "Synthetic Differential Geometry" ("SDG"), was not yet in general use; but the title of the book "Differential Geometry *without real numbers*" carries some insight which should not be forgotten. The title also indirectly points at the limitations of this kind of synthetic geometry: there is a vast amount of (differential-)geometric existence results that depend on the real number system, notably those that use *integrals*, say in the form of limits of Riemann sums.[1]

One aspect of the foundations of SDG which is used but not very much developed in the 1979 book, is the *contravariant* viewpoint, originating in French algebraic geometry in the 1950s. The covariant approach for the infinitesimal geometry of a space M resides mainly in the maps *into* M, like tangent vectors $D \to M$ (and hence are covariant in M), and making up the tangent bundle of M, whereas the contravariant approach of M resides in the *first neighbourhood of the diagonal* $M_{(1)} \subseteq M \times M$, determined by maps $M \to R$. The space $M_{(1)}$ is related to the cotangent bundle of M, and may be seen as building of the idea of Kähler (1930s) of "Kähler differentials" I/I^2. This approach was, from a synthetic perspective, considered already in [FMSTJB] (1979), and to a certian extent in the 1979 book. The contravariant viewpoint gives a more geometric perspective on the cotangent bundle, independent of a prior construction of the tangent bundle. The geometric perspective on differential forms (and several other differential-geometric notions) are developed further by several authors, and many of these developments may be found expounded in my [SGM] book (2010).

Thus, there are several things in this old book which we now can do more elegantly, and often better, than at that time.

So why should anybody be interested in the old book? One reason is that it contains many classical concrete things, which are not contained in the newer treatises, including, say, [SDG] (which, like other newer treatises, mainly address foundational issues); notably relating to contact geometry of the Euclidean plane,

[1] Synthetic differential geometry exploiting the real numbers $\mathbb{R}$, considering topos models constructed from $\mathbb{R}$, are discussed more deeply by Dubuc, by Moerdijk and Reyes, and in the recent book "*Synthetic Differential Topology*" London Math. Soc. Lecture Notes Series 448 , Cambridge University Press (2018) by Bunge, Gago and San Luis, to mention the most far-reaching.

like the action of vector fields on families of unparametrized curves. The classical texts on such issues are in some of Sophus Lie's writings, notably in the books "*Berührungstransformationen*" and ''*Differentialgleichungen*".

The book referred to in the Preface to the Second Edition, that "*I am presently writing for the Cambridge University Press*" is the item [SDG] in list below; it appeared in 1981. A Second Edition appeared in 2006.

[SDG]: A.K., *Synthetic Differential Geometry*, London Math. Soc. Lecture Notes Series 51, Cambridge University Press (1981); 2nd edition, London Math. Soc. Lecture Notes Series 333, Cambridge University Press (2006).
[FMSTJB]: A.K., Formal manifolds and synthetic theory of jet bundles", *Cahiers de Top. et Géom. Diff.* 21 (1980), 227-246
[SGM]: A.K., *Synthetic Geometry of Manifolds*, Cambridge Tracts in Mathematics 180, Cambridge University Press 2010.

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