Strong Functors and Monoidal Monads

By

Anders Kock

In [4] we proved that a commutative monad on a symmetric monoidal closed category carries the structure of a symmetric monoidal monad ([4], Theorem 3.2). We here prove the converse, so that, taken together, we have: there is a 1-1 correspondence between commutative monads and symmetric monoidal monads (Theorem 2.3 below).

The main computational work needed consists in constructing an equivalence between possible strengths

$$st_{A,B}\colon A \pitchfork B \to A\,T \pitchfork B\,T$$

on a functor, and possible "tensorial strengths" on T

$$t''_{X,B}\colon X \otimes B\,T \to (X \otimes B)\,T;$$

T is assumed to be a functor between categories tensored over a monoidal closed category $\mathscr{V}$. The equivalence is stated in Theorem 1.3. (There is a similar theorem for the notion of cotensorial strength $\lambda_{X,B}\colon (X \pitchfork B)\,T \to X \pitchfork B\,T$, which we do not include in this note.)

As an application of the theory here, we construct strength on certain functors related to the power set monad.

If $\mathscr{A}$ is a $\mathscr{V}$-category, we use $\pitchfork$ to denote the hom-functor $\mathscr{A}^{\mathrm{op}} \times \mathscr{A} \to \mathscr{V}$, as well as to denote the hom-functor of $\mathscr{V}$ itself.

1. Making a functor strong. Let $\mathscr{A}$ and $\mathscr{B}$ be categories tensored over the symmetric monoidal closed $\mathscr{V}$, [3]. Let $T\colon \mathscr{A}_0 \to \mathscr{B}_0$ be a functor between the underlying categories. To a family of maps

$$st_{A,A'}\colon A \pitchfork A' \to A\,T \pitchfork A'\,T \tag{1.1}$$

we associate a family of maps

$$t''_{X,A}\colon X \otimes A\,T \to (X \otimes A)\,T \tag{1.2}$$

by commutativity of

$$\begin{array}{ccc} X \otimes A\,T & \xrightarrow{\;t''_{X,A}\;} & (X \otimes A)\,T \\ {\scriptstyle u^A \otimes 1}\downarrow & & \uparrow{\scriptstyle ev} \\ (A \pitchfork (X \otimes A)) \otimes A\,T & \xrightarrow[st_{A,X\otimes A} \otimes 1]{} & (A\,T \pitchfork (X \otimes A)\,T) \otimes A\,T; \end{array} \tag{1.3}$$

conversely, to a family (1.2) we associate a family (1.1) by commutativity of

$$(1.4)\qquad \begin{array}{ccc} A \pitchfork A' & \xrightarrow{\quad st_{A,A'}\quad} & A\,T \pitchfork A'\,T \\ {\scriptstyle u^{AT}}\big\downarrow & & \big\uparrow{\scriptstyle 1\pitchfork (ev)\,T} \\ A\,T \pitchfork ((A \pitchfork A') \otimes A\,T) & \xrightarrow[1\pitchfork t''_{A\pitchfork A',A}]{} & A\,T \pitchfork ((A \pitchfork A') \otimes A)\,T\,. \end{array}$$

It is not difficult to prove that if the family (1.1) is natural (not necessarily $\mathscr{V}$-natural — we have not yet assumed that T is a $\mathscr{V}$-functor), then so is the family (1.2) constructed out of it; and if the family (1.2) is natural, then so is the family (1.1) constructed out of it. (To prove naturality of *st* in the first variable, as well as proving naturality of t'' in the second variable, involve diagrams consisting of seven naturality squares; whereas the remaining variables involve only three-square diagrams.)

Proposition 1.1. *The passages* (1.1) $\mapsto$ (1.2) *and* (1.2) $\mapsto$ (1.1) *are mutually inverse on natural families.*

Proof. Each argument consists in expanding the definitions, and chasing a diagram consisting of naturality squares (naturality of u, ev, and t'' in the one case; naturality of u, ev, and st in the other case) and some triangles expressing the adjunction equations between u and ev.

For any family *st* as in (1.1) we shall say that *st commutes with units* if

$$(1.5)\qquad \begin{array}{ccc} & I & \\ {\scriptstyle j_A}\swarrow & & \searrow{\scriptstyle j_{AT}} \\ A \pitchfork A & \xrightarrow[st]{} & A\,T \pitchfork A\,T \end{array}$$

commutes for all $A \in \mathscr{A}$. This diagram is the same as the diagram of Axiom VF 1′ in [2], p. 497. Likewise, we say that *st commutes with composition* if the diagram of Axiom VF 2′ (same place) commutes:

$$(1.6)\qquad \begin{array}{ccc} (B \pitchfork C) \otimes (A \pitchfork B) & \xrightarrow{\quad M\quad} & A \pitchfork C \\ {\scriptstyle st\otimes st}\big\downarrow & & \big\downarrow{\scriptstyle st} \\ (B\,T \pitchfork C\,T) \otimes (A\,T \pitchfork B\,T) & \xrightarrow[M]{} & A\,T \pitchfork C\,T\,. \end{array}$$

Proposition 1.2. *If the family st is natural and commutes with units and composition, then it makes T into a $\mathscr{V}$-functor $\bar T\colon \mathscr{A} \to \mathscr{B}$ with underlying functor $\bar T_0$ the original one. Conversely, the strength st of a $\mathscr{V}$-functor $\bar T$ is a family* (1.1) *which is natural with respect to the underlying functor T of $\bar T$, and which commutes with unit and composition.*

Proof. To prove the first part means just proving that $\bar T_0 = \bar T$, that is, for $a \in \mathscr{A}_0(A, A')$, we should prove

$$(a)\,T = (a)\,(st_{A,A'})\,V$$

(where $V\colon \mathscr{V} \to \mathscr{S}$ is part (ii) of the data of the closed category $\mathscr{V}$, see [2], I.2).

Using naturality of st with respect to a, this follows if

$$(1_A)\,(st_{A,A})\,V = 1_{AT};$$

but this holds since st commutes with units, and since $* \in (I)\,V$ by $(j_X)\,V$ is sent to 1_X for any X, by I.3.17 in [2].

Conversely, the strength of a $\mathscr{V}$-functor $\bar{T}$ commutes with units and composition by definition; and it is natural in both variables by Proposition I.9.4 in [2].

For any family t'' as in (1.2) we shall say that t'' *satisfies the unit condition* if

$$(1.7)\qquad \begin{array}{ccc} I \otimes A\,T & \xrightarrow{t''_{I,A}} & (I \otimes A)\,T \\ & {\scriptstyle l_{AT}}\searrow \cong & \cong \big\downarrow {\scriptstyle l_A T} \\ & & A\,T \end{array}$$

commutes for all $A \in \mathscr{A}$ (l_X being the isomorphism which is part of the data of $\mathscr{A}$ (resp. $\mathscr{B}$) being tensored over $\mathscr{V}$). Likewise, we say that t'' *satisfies the associativity condition* if

$$(1.8)\qquad \begin{array}{ccccc} X \otimes (Y \otimes A\,T) & \xrightarrow{1 \otimes t''} & X \otimes (Y \otimes A)\,T & \xrightarrow{t''} & (X \otimes (Y \otimes A))\,T \\ {\scriptstyle a}\big\downarrow \cong & & & & \cong \big\downarrow {\scriptstyle aT} \\ (X \otimes Y) \otimes A\,T & & \xrightarrow{\quad t'' \quad} & & ((X \otimes Y) \otimes A)\,T \end{array}$$

commutes for all $X, Y \in \mathscr{V}$, $A \in \mathscr{A}$ (here, the isomorphisms a are deducible from data for $\mathscr{A}$ (resp. $\mathscr{B}$) being tensored over $\mathscr{V}$; for $\mathscr{A} = \mathscr{B} = \mathscr{V}$, a is just the given associativity isomorphism for $\otimes$ in $\mathscr{V}$).

Theorem 1.3. *Let $\mathscr{A}$, $\mathscr{B}$ be categories tensored over $\mathscr{V}$, and let $T: \mathscr{A}_0 \to \mathscr{B}_0$ be a functor. Then the correspondence of Proposition 1.1 establishes a 1-1 correspondence between families st (as in (1.1)) making T into (the underlying of) a strong functor, and natural families t'' (as in (1.2)) which satisfy the unit and associativity condition.*

The theorem justifies calling t'' a *tensorial strength* on T.

Proof. A proof of the full theorem, as it stands, may be found in [5]. We shall here only give the proof for the case that $\mathscr{A} = \mathscr{B} = \mathscr{V}$, which is all we need for the main Theorem 2.3.

Let us start with t'', satisfying the conditions, in particular naturality; so, as we have remarked, the family st corresponding to it is natural. By Proposition 1.2 we need only check that st commutes with unit and composition.

To prove commutativity of (1.5), with st defined by (1.4) in terms of t'', we transpose the two legs of the diagram under the adjointness

$$(1.9)\qquad - \otimes A\,T \dashv A\,T \pitchfork -;$$

then j_{AT} yields

$$l_{AT}: I \otimes A\,T \xrightarrow{\cong} A\,T\,.$$

The composite $j_A \cdot st$, on the other hand, yields

$$j_A \otimes 1 \cdot st \otimes 1 \cdot ev = j_A \otimes 1 \cdot u^{AT} \otimes 1 \cdot (1 \pitchfork t'') \otimes 1 \cdot (1 \pitchfork (ev)\, T) \otimes 1 \cdot ev = \\ = j_A \otimes 1 \cdot u^{AT} \otimes 1 \cdot ev \cdot t'' \cdot (ev)\, T$$

by naturality of ev. Now $u^{AT} \otimes 1$ and ev cancel, by adjunction equations, whence we are left with

$$\begin{aligned} & j_A \otimes 1 \cdot t'' \cdot (ev)\, T = \\ & = t'' \cdot (j_A \otimes 1)\, T \cdot (ev)\, T = \\ & = t'' \cdot l_A\, T \end{aligned}$$

by naturality of t'' with respect to j_A, and by the definition of l_A as transpose of j_A. But $t'' \cdot l_A T$ is l_{AT}, by the assumption (1.7).

To prove that st commutes with composition, transpose both legs of (1.6) under the adjointness (1.9), with st expanded in terms of t''. Using naturality of ev and of t'', it is easy to see that the clockwise composite yields (1.10) (where we for ease of notation assume $\otimes$ to be strictly associative)

$$(1.10) \qquad \begin{aligned} & (B \pitchfork C) \otimes (A \pitchfork B) \otimes A\, T \xrightarrow{t''} ((B \pitchfork C) \otimes (A \pitchfork B) \otimes A)\, T \to \\ & \xrightarrow{(M \otimes 1)\, T} ((A \pitchfork C) \otimes A)\, T \xrightarrow{(ev)\, T} C\, T\,. \end{aligned}$$

The transpose of the counterclockwise composite of (1.6), on the other hand, is

$$st \otimes st \otimes 1_{AT} \cdot M \otimes 1_{AT} \cdot ev$$

which by Lemma 1.3 in [4] (which says just $M \otimes 1 \cdot ev = 1 \otimes ev \cdot ev$) is

$$st \otimes st \otimes 1_{AT} \cdot 1 \otimes ev \cdot ev = st \otimes 1 \otimes 1 \cdot 1 \otimes st \otimes 1 \cdot 1 \otimes ev \cdot ev\,.$$

From the construction of st in terms of t'', it is obvious that this equals

$$\begin{aligned} & st \otimes 1 \otimes 1 \cdot 1 \otimes t'' \cdot 1 \otimes (ev)\, T \cdot ev = \\ & = 1 \otimes t'' \cdot 1 \otimes (ev)\, T \cdot st \otimes 1 \cdot ev = \\ & = 1 \otimes t'' \cdot 1 \otimes (ev)\, T \cdot t'' \cdot (ev)\, T \end{aligned}$$

the last equation again by definition of the relation between st and t''. Finally, using naturality of t'', we get

$$1 \otimes t'' \cdot t'' \cdot (1 \otimes ev)\, T \cdot (ev)\, T$$

which, again by Lemma 1.3 in [4], is

$$1 \otimes t'' \cdot t'' \cdot (M \otimes 1)\, T \cdot (ev)\, T,$$

which by the assumed associativity condition (1.8) for t'' equals (1.10). This proves that st commutes with composition M. — Let us remark that, in the proof of the theorem in its full strength, the "associativity" of the tensor product which makes $\mathscr{A}$ (or $\mathscr{B}$) tensored over $\mathscr{V}$, is not given as a primitive, but has to be constructed; consequently, the Lemma 1.3 of [4], which we used twice, must be replaced in the above argument, by an analogous (but not so easily proved) relation between composition and evaluation.

Conversely, if st is a strength, the corresponding t'' satisfies the unit and associativity condition, by Proposition 1.5 in [4]. This proves the theorem (in the case $\mathscr{A} = \mathscr{B} = \mathscr{V}$).

Remark 1.4. Suppose $T_0, T_1 : \mathscr{A} \to \mathscr{B}$ are functors with $t_0'' : X \otimes B\,T_0 \to (X \otimes B)\,T_0$ derived from a strength st_0 of T_0, and similarly t_1'' derived from a strength st_1 of T_1. Then a family

$$\tau_A : A\,T_0 \to A\,T_1$$

is $\mathscr{V}$-natural if and only if

$$\begin{array}{ccc} X \otimes B\,T_0 & \xrightarrow{t_0''} & (X \otimes B)\,T_0 \\ {\scriptstyle 1\otimes\tau_B}\downarrow & & \downarrow{\scriptstyle \tau_{X\otimes B}} \\ X \otimes B\,T_1 & \xrightarrow[t_1'']{} & (X \otimes B)\,T_1 \end{array}$$

commutes for all X, B. This is quite easy to see.

Let again $\mathscr{A}, \mathscr{B}, \mathscr{C}$ be tensored over $\mathscr{V}$. If $T_0 : \mathscr{A} \to \mathscr{B}$, $T_1 : \mathscr{B} \to \mathscr{C}$ are $\mathscr{V}$-functors, then $T_0 \cdot T_1 : \mathscr{A} \to \mathscr{C}$ carries a canonical "composite" strength. If t_0'', t_1'' are the tensorial strengths, then the tensorial strength of $T_0 \cdot T_1$ corresponding to the composite strength is given by

$$X \otimes (A)\,T_0\,T_1 \xrightarrow{t_1''} (X \otimes A\,T_0)\,T_1 \xrightarrow{(t_0'')T_1} (X \otimes A)\,T_0\,T_1\,.$$

The proof of this is formally the same as the proof of Lemma 1.2 in [4].

2. Making a monoidal functor strong. The results of section 1 apply in particular to functors $T : \mathscr{V}_0 \to \mathscr{V}_0$, where $\mathscr{V}$ is a monoidal closed category. Recall [1], or [2], p. 473—474, that making T into a *monoidal* functor means giving a natural

$$\psi_{A,B} : A\,T \otimes B\,T \to (A \otimes B)\,T$$

and a map

$$\psi^0 : I \to I\,T\,,$$

satisfying unit and associativity conditions (MF1—MF3, p. 473 in [2]). A transformation between monoidal functors is *monoidal,* if it is compatible with ψ, ψ^0 (MN1, MN2, p. 474 in [2]). The identity functor $1 : \mathscr{V}_0 \to \mathscr{V}_0$ carries a canonical (identity) monoidal structure. The composite of two monoidal functors carries a "composite" monoidal structure.

Proposition 2.1. *Let* T, ψ, ψ^0 *be a monoidal functor* $\mathscr{V} \to \mathscr{V}$. *Let* $\eta : 1 \Rightarrow T$ *be a monoidal transformation. Then the composites* ($A, B \in \mathscr{V}$):

$$(2.0) \qquad A \otimes B\,T \xrightarrow{\eta_A \otimes 1} A\,T \otimes B\,T \xrightarrow{\psi_{A,B}} (A \otimes B)\,T$$

constitute a tensorial strength t'' *for* T.

Proof. From MN1 it follows that $\eta_I = \psi^0$; the unit condition for t'' is then MF1 for T, ψ, ψ^0. The associativity condition is easily proved by a small diagram chase using MN2 for η and MF3 for T, ψ, ψ^0.

It is easy to see that if $\tau\colon T \Rightarrow T'$ is a monoidal transformation between monoidal functors, and $\eta\colon 1 \Rightarrow T$ is monoidal, then the two monoidal transformations

$$\eta\colon 1 \Rightarrow T\,, \qquad \eta \cdot \tau\colon 1 \Rightarrow T'$$

give rise to tensorial strengths with respect to which τ is a *strong* natural transformation. In particular, if $((T, \psi, \psi^0), \eta, \mu)$ is a *monoidal* monad on $\mathscr{V}$ (meaning that η and μ are monoidal transformations), then the strength on T derived from η makes (T, η, μ) into a *strong* monad; for, η and μ will be strong transformations since the diagrams

$$\begin{array}{ccc} & & 1 \\ & {\scriptstyle 1}\nearrow & \\ 1 & & \Big\downarrow{\scriptstyle \eta} \\ & {\scriptstyle \eta}\searrow & \\ & & T \end{array} \qquad\qquad \begin{array}{ccc} & & T^2 \\ & {\scriptstyle \eta T}\nearrow & \\ & T & \\ {\scriptstyle \eta}\nearrow & & \Big\downarrow{\scriptstyle \mu} \\ 1 & & \\ & {\scriptstyle \eta}\searrow & \\ & & T \end{array}$$

commute, and since the monoidal transformation $\eta \cdot \eta T$ is easily seen to give rise to the "iterated" tensorial strength $t'' \cdot t'' T$ on T^2.

We now assume (for the first time in this paper) a symmetry

$$c_{A,B}\colon A \otimes B \to B \otimes A$$

given on $\mathscr{V}$. Then, by [4], to a strong monad $((T, t''), \eta, \mu)$ there exist *two* monoidal structures on T

(2.1) $$\overline{\psi}\colon A\,T \otimes B\,T \xrightarrow{t'} (A \otimes B\,T)\,T \xrightarrow{t''T} (A \otimes B)\,T^2 \xrightarrow{\mu} (A \otimes B)\,T$$

and

(2.2) $$\tilde{\psi}\colon A\,T \otimes B\,T \xrightarrow{t''} (A\,T \otimes B)\,T \xrightarrow{t'T} (A \otimes B)\,T^2 \xrightarrow{\mu} (A \otimes B)\,T\,.$$

where $t' = c \cdot t'' \cdot c\,T$. If the strong monad was derived, as above, from a monoidal monad $((T, \psi, \psi^0), \eta, \mu)$ one may ask: when is $\overline{\psi}$ or $\tilde{\psi}$ equal to ψ? A partial answer is given by

Proposition 2.2. *If $((T, \psi, \psi^0), \eta, \mu)$ is a monoidal monad and (T, ψ, ψ^0) is a symmetric monoidal functor meaning* ([2], MF4) *that the following diagram commutes*

(2.3) $$\begin{array}{ccc} A\,T \otimes B\,T & \xrightarrow{\psi} & (A \otimes B)\,T \\ {\scriptstyle c}\Big\downarrow & & \Big\downarrow{\scriptstyle cT} \\ B\,T \otimes A\,T & \xrightarrow[\psi]{} & (B \otimes A)\,T\,, \end{array}$$

then $\psi = \overline{\psi} = \tilde{\psi}$.

Proof. From the symmetry condition (2.3), and from $c \cdot c = 1$, it is immediate that $t' = c \cdot t'' \cdot c\,T$ (with $t'' = \eta \otimes 1 \cdot \psi$) may be described directly as $1 \otimes \eta \cdot \psi$. Then

(2.4) $$\begin{aligned} \overline{\psi} &= t' \cdot t''\,T \cdot \mu = 1 \otimes \eta \cdot \psi \cdot (\eta \otimes 1)\,T \cdot \psi\,T \cdot \mu = \\ &= 1 \otimes \eta \cdot \eta\,T \otimes 1 \cdot \psi \cdot \psi\,T \cdot \mu\,. \end{aligned}$$

the last equality sign just by naturality of ψ. But now, the assumption that μ is a monoidal transformation says precisely

$$\psi \cdot \psi T \cdot \mu = \mu \otimes \mu \cdot \psi; \tag{2.5}$$

$\psi = \bar{\psi}$ is immediate from (2.4), (2.5), and monad laws. The proof of $\psi = \tilde{\psi}$ is similar.

Theorem 2.3. *Let T, η, μ be a monad on the underlying category $\mathscr{V}_0$ of a symmetric monoidal closed category $\mathscr{V}$. Then there is a 1-1 correspondence between the following two kinds of structure on T:*

(i) *a strength st on T making $((T, st), \eta, \mu)$ into a commutative monad.*

(ii) *a monoidal structure ψ, ψ^0 on T making $((T, \psi, \psi^0), \eta, \mu)$ into a symmetric monoidal monad.*

Proof. Starting with the symmetric monoidal structure, Proposition 2.2 asserts that the tensorial strength constructed makes the monad commutative, and that it gives ψ, ψ^0 back by the described process. Combining this fact with Theorem 1.3 tells us that the processes $(\psi, \psi^0) \mapsto st \mapsto (\psi, \psi^0)$ give the original monoidal structure back. Conversely, starting with a commutative monad, the process gives, by Theorem 3.2 of [4] rise to a symmetric monoidal structure ψ, ψ^0 with $\psi^0 = \eta_I$. The tensorial strength $\tilde{t}''_{A,B}$ constructed out of $\psi = t' \cdot t'' T \cdot \mu$ is

$$\begin{aligned}\eta_A \otimes 1 \cdot \psi_{A,B} &= \eta_A \otimes 1 \cdot t'_{A,BT} \cdot t''_{A,B} T \cdot \mu = \\ &= \eta_{A \otimes BT} \cdot t''_{A,B} T \cdot \mu = t''_{A,B} \cdot \eta_{(A \otimes B)T} \cdot \mu = \\ &= t''_{A,B}\end{aligned}$$

using the definition of ψ, the unit law for t', naturality of η, and a monad law, respectively. Thus the two processes give the original t'' back. Combining this fact with Theorem 1.3 tells us that the processes $(st) \mapsto (\psi, \psi^0) \mapsto (st)$ give the original strength back. This proves the theorem.

Example. Let $\mathscr{E}$ be an elementary topos in the sense of Lawvere and Tierney, [6]. They proved that the assignment

$$A \mapsto A \pitchfork \Omega$$

(where Ω is the recipent object for characteristic functions) becomes a covariant functor P by letting $(f)P$ be the left adjoint of $f \pitchfork \Omega$. If $\mathscr{E}$ = sets, P is the power-set functor. It is easy to make P into a monoidal functor, in fact, by the "product subset" construction; let

$$\psi: A P \times B P \to (A \times B) P$$

be the map whose transpose $(A P \times B P) \times (A \times B) \to \Omega$ is the characteristic function for

$$\varepsilon_A \times \varepsilon_B \to ((A \pitchfork \Omega) \times A) \times ((B \pitchfork \Omega) \times B) \cong (A P \times BP) \times (A \times B)\,. \tag{2.5}$$

(ε_X is the subobject whose characteristic function is the evaluation $(X \pitchfork \Omega) \times X \to \Omega$.) Then ψ is a right adjoint for

$$\varkappa: (A \times B) P \to A P \times B P$$

(defined by $\varkappa \cdot proj_i = (proj_i)\, P$, $i = 0,1$). Since $\varkappa$ satisfies an obvious associativity condition, we immediately get, by passing to right adjoints, the associativity condition required for P to be a monoidal functor via ψ (Axiom MF3 in [2]). For ψ^0: $1 \to 1\,P = \Omega$ we take the maximal map $t\colon 1 \to \Omega$. Since the (only) map $\Omega \to 1$ is a left adjoint for t, the unit conditions for (P, ψ, ψ^0) are again proved by passing to adjoints.

The "singleton" transformation

$$\mathrm{id} \xrightarrow{\eta} P$$

defined by letting $\eta_A : A \to A\,P = A \pitchfork \Omega$ be the transpose of the characteristic function of the diagonal $A \to A \times A$ can be proved monoidal by the technique characteristic for elementary toposes: by comparing two maps into $X \pitchfork \Omega$, pass by exponential adjointness to two maps into Ω, and prove that the two subobjects classified by these maps are equal. Specifically, to prove η monoidal means proving that two maps $A \times B \to (A \times B) \pitchfork \Omega$ agree. By the procedure described, we end up by proving that two certain subobjects of $(A \times B) \times (A \times B)$ are equal, namely in fact both the diagonal $A \times B \to (A \times B) \times (A \times B)$.

References

[1] J. BENABOU, Catégories avec multiplication. C. R. Acad. Sci. Paris **256**, 1887–1890 (1963).

[2] S. EILENBERG and M. KELLY, Closed Categories. In: Proc. Conf. Categorical Algebra, La Jolla 1965. Berlin 1966.

[3] M. KELLY, Adjunction for enriched categories. Reports of the Midwest Category Seminar III. Lecture Notes in Math. **106**, 166–177, Berlin 1969.

[4] A. KOCK, Monads on symmetric monoidal closed categories. Arch. Math. **21**, 1–10 (1970).

[5] A. KOCK, Strong Functors and monoidal monads. Århus Universitet, Various Publications Series Nr. **11** (1970).

[6] F. W. LAWVERE, Quantifiers and Sheaves. Actes Congrès International Math. (Nice) 1970, Tome 1, p. 329–334.

Eingegangen am 26. 3. 1971

Anschrift des Autors:

Anders Kock
Matematisk Institut
Aarhus Universitet
8000 Aarhus C, Danmark